TFT construction of CFT correlators beyond semisimplicity

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joint work with Ingo Runkel
(partially based on [2405.18038])

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CFT: Algebraic, Topological and Probabilistic approaches in Conformal Field Theory
Institute Pascal – Orsay

October 08, 2025

Motivation

"Symmetry topological field theory framework"

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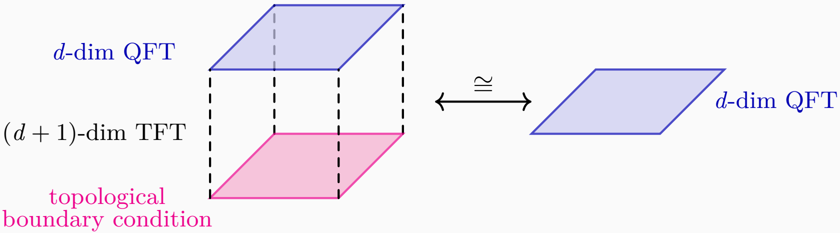

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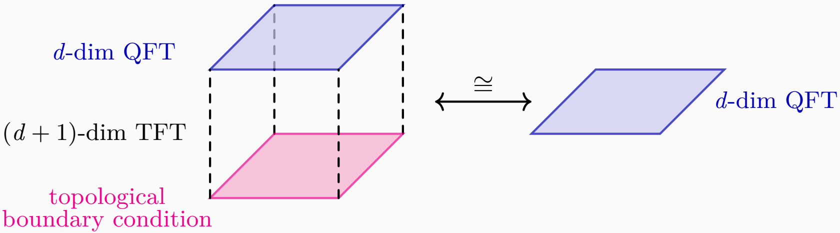

Use these ideas to understand full 2d CFTs!

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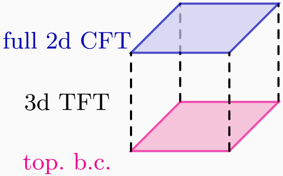

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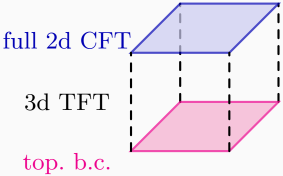

Chiral CFT

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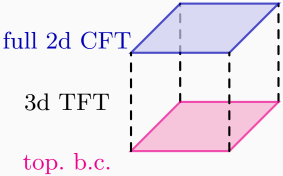

Chiral CFT: (i) VOA $\mathcal{V}$

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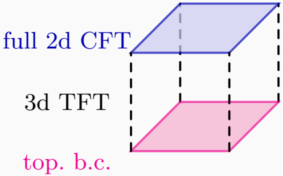

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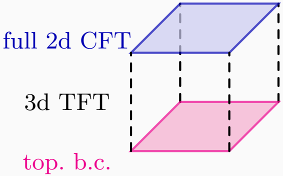

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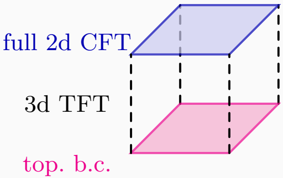

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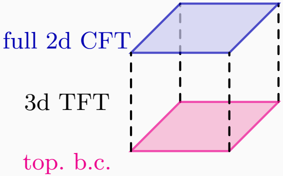

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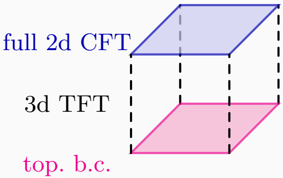

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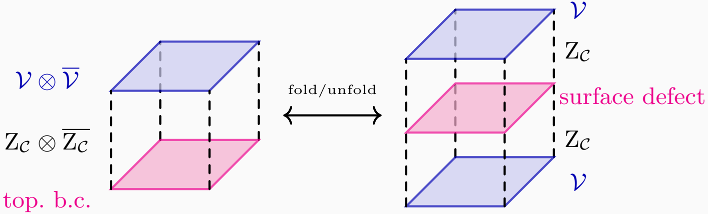

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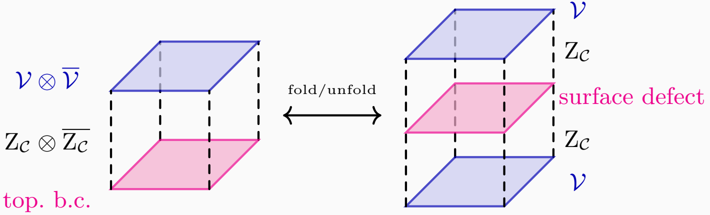

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For semisimple/rational chiral CFTs, surface defects in the 3d TFTs of [RT91] constructed from Rep($\mathcal{V}$) give all possible full CFTs.

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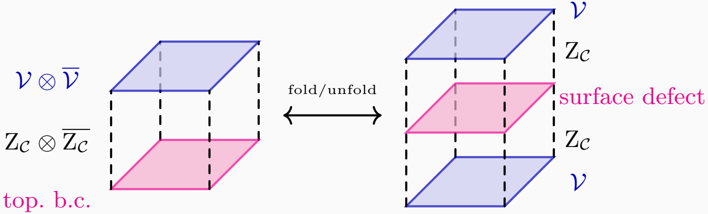

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Goal: Use surface defects in the 3d TFTs of [DGGPR22] to obtain full CFTs in the non-semisimple/logarithmic setting.

Modular functors from non-semisimple TFTs

Chiral modular functors from non-semisimple TFTs

Theorem [H., Runkel]

For $\mathcal{C}$ a not necessarily semisimple modular tensor category, the 3d TFT $\mathrm{Z}_{\mathcal{C}}$ of [DGGPR22] induces a symmetric monoidal 2-functor:

$$\mathrm{Bl}^{\chi}_{\mathcal{C}} \colon \mathrm{Bord}^{\chi}_{2+\varepsilon,2,1} \longrightarrow \mathcal{L}\mathrm{ex}_{\Bbbk} \qquad (\text{finite version of } \widehat{\mathbb{C}}\mathrm{at}_{\Bbbk})$$

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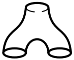
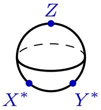
 $\longmapsto \Big((X, Y, Z) \mapsto \mathrm{Z}_{\mathcal{C}}\big($  $\big)\Big)$

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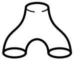

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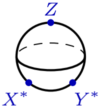

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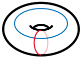

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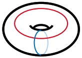

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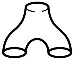

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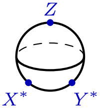

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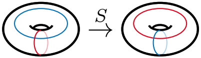

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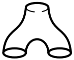
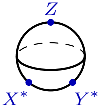
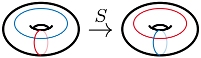
 $\longmapsto \Big((X, Y, Z) \mapsto \mathrm{Z}_\mathcal{C}\Big($  $\Big)\Big)$

 $\longmapsto \Big(\mathrm{Z}_\mathcal{C}(T^2) \xrightarrow{\mathrm{Z}_\mathcal{C}((T^2\times I)_S)} \mathrm{Z}_\mathcal{C}(T^2) \Big)$

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Remark: Together with [DGGPR23], this provides a 3d construction of Lyubashenko's modular functor [Lyu95].

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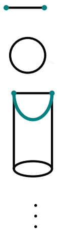
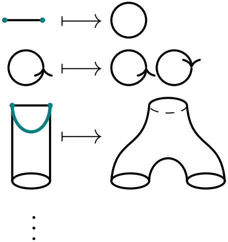


Full modular functors from non-semisimple TFTs

Definition/Lemma

There is an open-closed bordism 2-category, and a symmetric monoidal 2-functor:

$$\widehat{(-)}\colon \mathrm{Bord}^{\mathrm{oc}}_{2+\varepsilon,2,1} \longrightarrow \mathrm{Bord}^{\chi}_{2+\varepsilon,2,1}$$



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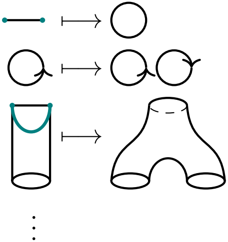

From now on: $\mathrm{Bl}_{\mathcal{C}} := \mathrm{Bl}^{\chi}_{\mathcal{C}} \circ \widehat{(-)}$

Full CFTs from surface defects

Theorem [H., Runkel]

Every surface defect A in the 3d TFT $Z_{\mathcal{C}}$, satisfying some technical assumptions, induces a braided monoidal 2-natural transformation:

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+ compatibility conditions for gluing surfaces and with mapping class group actions;

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[FRS02]: For Σ a 2-manifold get $\mathrm{Cor}_{\Sigma}^{A}$ via the "connecting bordism":

$$M_{\Sigma} := \Sigma \times [-1,1]/\sim \quad \text{with} \quad (p,t) \sim (p,-t) \quad \text{for} \quad p \in \partial^{\mathrm{ph}}\Sigma$$

with A-labelled surface defect Σ at $\Sigma \times \{0\}$.

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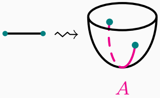

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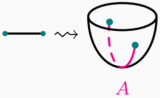

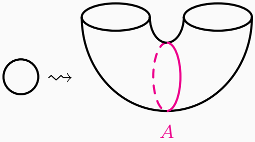

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For S a 1- or 2-manifold get Cor_S^{A} via the "connecting manifold":

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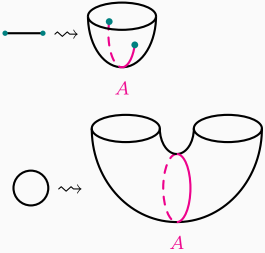

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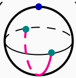
$\longrightarrow \Big(X \mapsto Z_{\mathcal{C}}\Big($ 

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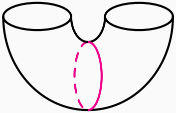
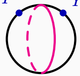
 $\rightsquigarrow$ 

$\longrightarrow \Big((Y, \overline{Y}) \mapsto Z_{\mathcal{C}}\Big($ 

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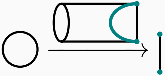
Example $\Sigma :$ 

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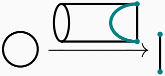
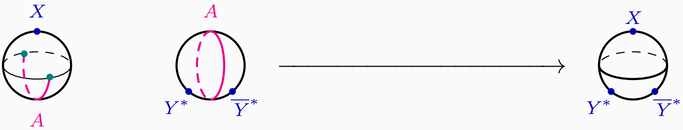
Example Σ : 



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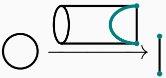
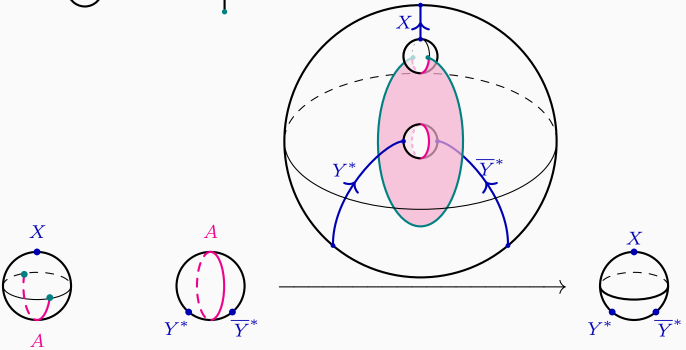
Example Σ : 



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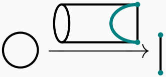
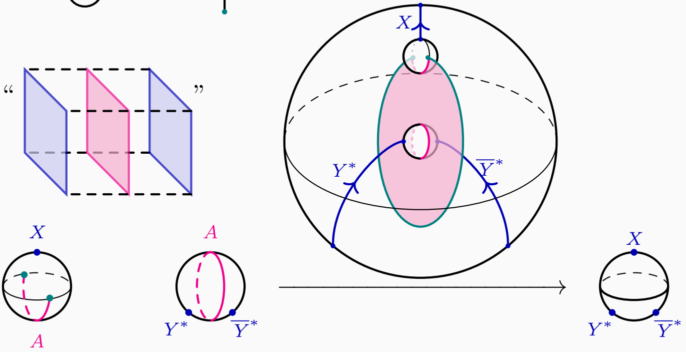
Example Σ : 



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Our main technical contribution to this theorem is the following result on the behaviour of $\mathrm{Bl}_{\mathcal{C}}^{\chi}$ under gluing:

Proposition

Let Σ be a (possibly connected) surface with at least one incoming and outgoing boundary component, and let Σ_{gl} be the surface obtained from gluing these boundaries. Then there is a natural isomorphism

$$\mathrm{Bl}_{\mathcal{C}}^{\chi}(\Sigma_{\mathrm{gl}}) \cong \oint^{X \in \mathcal{C}} \mathrm{Bl}_{\mathcal{C}}^{\chi}(\Sigma)(X, X).$$

induced by a 3-dimensional bordism.

Chiral modular functors from non-semisimple TFTs

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